
AP[®] Calculus AB

Sample Student Responses and Scoring Commentary

Inside:

Free-Response Question 6

- ☒ **Scoring Guidelines**
- ☒ **Student Samples**
- ☒ **Scoring Commentary**

Part B (AB): Graphing calculator not allowed**Question 6****9 points****General Scoring Notes**

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Consider the curve G defined by the equation $y^3 - y^2 - y + \frac{1}{4}x^2 = 0$.

	Model Solution	Scoring
A	Show that $\frac{dy}{dx} = \frac{-x}{2(3y^2 - 2y - 1)}$.	
	$\frac{d}{dx}\left(y^3 - y^2 - y + \frac{1}{4}x^2\right) = \frac{d}{dx}(0)$ $\Rightarrow 3y^2 \cdot \frac{dy}{dx} - 2y \cdot \frac{dy}{dx} - \frac{dy}{dx} + \frac{x}{2} = 0$	Implicit differentiation Point 1 (P1)
	$\Rightarrow (3y^2 - 2y - 1)\frac{dy}{dx} = -\frac{x}{2}$ $\Rightarrow \frac{dy}{dx} = \frac{-x}{2(3y^2 - 2y - 1)}$	Verification Point 2 (P2)
Scoring Notes for Part A		

- P1** is earned only for the correct implicit differentiation of $y^3 - y^2 - y + \frac{1}{4}x^2 = 0$. Responses may use alternative notations for $\frac{dy}{dx}$, such as y' .
- To be eligible for **P2**, a response must have earned **P1**.
- It is sufficient to present $(3y^2 - 2y - 1)\frac{dy}{dx} = -\frac{x}{2}$ to earn **P2**, provided there are no subsequent errors.

- B** There is a point P on the curve G near $(2, -1)$ with x -coordinate 1.6. Use the line tangent to the curve at $(2, -1)$ to approximate the y -coordinate of point P .

$\left. \frac{dy}{dx} \right _{(x,y)=(2,-1)} = \frac{-2}{2(3+2-1)} = -\frac{1}{4}$	Slope of tangent line	Point 3 (P3)
$y \approx -1 - \frac{1}{4}(1.6 - 2) = -0.9$	Tangent line approximation	Point 4 (P4)

Scoring Notes for Part B

- A response can earn **P3** with $\left. \frac{dy}{dx} \right|_{(x,y)=(2,-1)} = -\frac{1}{4}$, $\frac{dy}{dx} = -\frac{1}{4}$, “slope is $-\frac{1}{4}$,” or equivalent.
- A response that presents a linear approximation with a slope of $-\frac{1}{4}$ also earns **P3**.
- A response that declares $\left. \frac{dy}{dx} \right|_{(x,y)=(2,-1)}$ (or the slope) equal to any nonzero value $k \neq -\frac{1}{4}$ does not earn **P3**. Such a response earns **P4** for a presented approximation mathematically equivalent to $-1 + k(-0.4)$.
- P4** cannot be earned with a linear approximation using a slope other than $-\frac{1}{4}$ if that slope has not been declared to be the value of $\left. \frac{dy}{dx} \right|_{(x,y)=(2,-1)}$.
- A response does not have to present the tangent line equation but must clearly demonstrate its use at $x = 1.6$ in finding the requested approximation to be eligible for **P4**.
- A response of $-1 - \frac{1}{4}(-0.4)$ earns both **P3** and **P4**.
- A response of $-1 - \frac{1}{4}(1.6 - 2)$ or equivalent earns **P4** (i.e., subsequent errors in simplification will not be considered in scoring for **P4**).

Note: An ambiguous response, such as $-1 - \frac{1}{4}(1.6 - 2)$, does not earn **P4** and therefore must go on to resolve the ambiguity with a correct final answer (e.g., -0.9) to earn **P4**.

- C** For $x > 0$ and $y > 0$, there is a point S on the curve G at which the line tangent to the curve at that point is vertical. Find the y -coordinate of point S . Show the work that leads to your answer.

For $x > 0$, the curve G has a vertical tangent line when
 $2(3y^2 - 2y - 1) = 0$.

Sets denominator equal to 0 **Point 5 (P5)**

$$2(3y^2 - 2y - 1) = 0 \Rightarrow 2(3y + 1)(y - 1) = 0$$

Because $y > 0$, it follows that $y = 1$.

The line tangent to the curve is vertical at the point on the curve where $y = 1$.

Answer **Point 6 (P6)**

Scoring Notes for Part C

- **P5** is earned with any of $2(3y^2 - 2y - 1) = 0$, $3y^2 - 2y - 1 = 0$, $2(3y \pm 1)(y \pm 1) = 0$, or $(3y \pm 1)(y \pm 1) = 0$.
- To be eligible for **P6**, a response must have earned **P5**.
- A response does not need to consider the numerator of $\frac{dy}{dx} = \frac{-x}{2(3y^2 - 2y - 1)}$ to earn **P5** or **P6**; considering the denominator is sufficient.
- A response that states solutions of $y = -\frac{1}{3}$ and $y = 1$, but does not clearly identify that $y = 1$ is the only solution to the prompt, does not earn **P6**.
- In the presence of algebraic work to find the value of y , **P6** is earned only if the algebraic work is correct.
- A response of “ $2(3y^2 - 2y - 1) = 0$, $y = 1$ ” earns both **P5** and **P6**.

- D** A particle moves along the curve H defined by the equation $2xy + \ln y = 8$. At the instant when the particle is at the point $(4, 1)$, $\frac{dx}{dt} = 3$. Find $\frac{dy}{dt}$ at that instant. Show the work that leads to your answer.

$\frac{d}{dt}(2xy + \ln y) = \frac{d}{dt}(8)$ $2\frac{dx}{dt}y + 2x\frac{dy}{dt} + \frac{1}{y}\frac{dy}{dt} = 0$	Attempts implicit differentiation with respect to t	Point 7 (P7)
	$2\frac{dx}{dt}y + 2x\frac{dy}{dt} + \frac{1}{y}\frac{dy}{dt} = 0$	Point 8 (P8)
$2(3)(1) + 2(4)\frac{dy}{dt} + \frac{1}{1}\frac{dy}{dt} = 0$ $\Rightarrow 6 + 9\frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = -\frac{2}{3}$	Answer	Point 9 (P9)

Scoring Notes for Part D

- P7** is earned for implicitly differentiating $2xy + \ln y = 8$ with respect to t with at most one error.
- P8** is earned for an equation equivalent to $2\frac{dx}{dt}y + 2x\frac{dy}{dt} + \frac{1}{y}\frac{dy}{dt} = 0$.
- To be eligible for **P9**, a response must have earned **P7** and **P8**, with no errors in implicit differentiation.
- P9** is earned only for the value of $-\frac{2}{3}$.
- Alternate solution:

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

$$\frac{d}{dx}(2xy + \ln y) = \frac{d}{dx}(8) \Rightarrow 2y + 2x\frac{dy}{dx} + \frac{1}{y}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-2y}{2x + \frac{1}{y}}$$

$$\left. \frac{dy}{dx} \right|_{(x,y)=(4,1)} = \frac{-2(1)}{2(4) + \frac{1}{1}} = -\frac{2}{9}$$

$$\left. \frac{dy}{dt} \right|_{(x,y)=(4,1)} = \left. \frac{dy}{dx} \cdot \frac{dx}{dt} \right|_{(x,y)=(4,1)} = -\frac{2}{9} \cdot 3 = -\frac{2}{3}$$

- P7** is earned for an implicit differentiation with respect to x with at most one error, as long as $\frac{dy}{dx}$ is eventually correctly linked to $\frac{dx}{dt}$.
- P8** is earned for finding $\frac{dy}{dx}$ and multiplying the result by $\frac{dx}{dt} = 3$.
- P8** can be earned for a stated incorrect $\frac{dy}{dx}$, as long as it is multiplied by 3.
- P9** is only earned for a correct value of $-\frac{2}{3}$ or equivalent.
- Stating $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$ alone does not earn any points.

Q6



NO CALCULATOR ALLOWED

Q6

Answer QUESTION 6 PARTS A and B on this page.

PART A

$$y^3 - y^2 - y + \frac{1}{4}x^2 = 0$$

$$y^3 - y^2 - y = -\frac{1}{4}x^2$$

$$3y^2 \frac{dy}{dx} - 2y \frac{dy}{dx} - \frac{dy}{dx} = -\frac{1}{2}x$$

$$\frac{dy}{dx} (3y^2 - 2y - 1) = -\frac{1}{2}x$$

$$\frac{dy}{dx} = \frac{-x}{2(3y^2 - 2y - 1)} \checkmark$$

PART B

$$\frac{dy}{dx} = \frac{-x}{2(3y^2 - 2y - 1)} = \frac{-2}{2(3(-1)^2 - 2(-1) - 1)} = \frac{-2}{2(3+2-1)} = \frac{-2}{8} = -\frac{1}{4}$$

$$y - (-1) = -\frac{1}{4}(x - 2)$$

$$y + 1 = -\frac{1}{4}x + \frac{1}{2}$$

$$y = -\frac{1}{4}x - \frac{1}{2}$$

$$y \approx -\frac{1}{4}\left(\frac{16}{10}\right) - \frac{1}{2}$$

$$y \approx -\frac{4}{10} - \frac{1}{2}$$

$$\approx -\frac{4}{10} - \frac{5}{10} = \boxed{-\frac{9}{10}} \text{ or } \boxed{-0.9}$$

Page 14

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

0129349



Q6

NO CALCULATOR ALLOWED

Q6

Answer QUESTION 6 PARTS C and D on this page.

PART C

$$2(3y^2 - 2y - 1) = 0$$

$$3y^2 - 2y - 1 = 0$$

$$3y^2 - 3y + y - 1 = 0$$

$$3y(y-1) + 1(y-1) = 0$$

$$(3y+1)(y-1) = 0$$

$$3y+1=0 \quad y-1=0$$

$$y = \frac{-1}{3} \quad y = 1$$

The y-coordinate is 1 $\rightarrow$ $y = 1$ **PART D**

$$(2x)\left(\frac{dy}{dt}\right) + 2\left(\frac{dx}{dt}\right)(y) + \frac{1}{y}\frac{dy}{dt} = 0$$

$$2(4)\frac{dy}{dt} + 2(3)(1) + \frac{1}{1}\frac{dy}{dt} = 0$$

$$8\frac{dy}{dt} + 6 + \frac{dy}{dt} = 0$$

$$9\frac{dy}{dt} + \frac{dy}{dt} = -6$$

$$\frac{dy}{dt}(9+1) = -6$$

$$\frac{dy}{dt} = \frac{-6}{9} = \boxed{-\frac{2}{3}}$$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Q6

 NO CALCULATOR ALLOWED

Q6

Answer QUESTION 6 PARTS A and B on this page.

PART A

$$y^3 - y^2 - y + \frac{1}{4}x^2 = 0$$

$$3y^2 \frac{dy}{dx} - 2y \frac{dy}{dx} - \frac{dy}{dx} + \frac{1}{2}x = 0$$

$$(3y^2 - 2y - 1) \frac{dy}{dx} = -\frac{x}{2}$$

$$\frac{dy}{dx} = \frac{-x}{2(3y^2 - 2y - 1)}$$

PART B

$$y+1 = -\frac{1}{4}(x-2)$$

$$\frac{dy}{dx} = \frac{-2}{2(3(-1)^2 - 2(-1) - 1)} = \frac{-2}{2(3+2-1)} = -\frac{2}{8} = -\frac{1}{4}$$

$$-\frac{1}{4}x + \frac{1}{2} - \frac{2}{2} = \left(-\frac{1}{4}x - \frac{1}{2}\right)^4$$

$$-1.6 - 2 = -3.6$$

$$-x - 2 = -$$

Page 14

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

0196176



Q6



NO CALCULATOR ALLOWED

Q6

Answer QUESTION 6 PARTS C and D on this page.

PART C

$$\frac{dy}{dx} = \frac{-x}{2(3y^2 - 2y - 1)} \rightarrow \frac{1}{0}$$

$$2(3y^2 - 2y - 1) = 0$$

$$6y^2 - 4y - 2 = 0$$

$$2(3y^2 - 2y - 1)$$

$$3y^2 - 3y + y - 1$$

$$3y(y-1) + 1(y-1)$$

$$3y+1 \quad y-1$$

$$\cancel{y = -\frac{1}{3}} \quad y = 1$$

PART D

$$2xy + \ln y = 8$$

$$y = 1 \quad \frac{dx}{dt} = 3$$

$$x = 4$$

$$2x \frac{dy}{dt} + y \cdot 2 \frac{dx}{dt} + \frac{1}{y} \frac{dy}{dt} = 0$$

$$\frac{dy}{dt} (2x + \frac{1}{y}) + 2(1)(3)$$

$$\frac{dy}{dt} (2(4) + 1) + 2(1)(3) = 14$$

$$\frac{dy}{dx} = \left(\frac{1}{54} \right)$$

Q6



NO CALCULATOR ALLOWED

Q6

Answer QUESTION 6 PARTS A and B on this page.

PART A

$$3y^2 \frac{dy}{dx} - 2y \frac{dy}{dx} - 1 \frac{dy}{dx} + \frac{1}{2}x = 0$$

$$\frac{dy}{dx} (3y^2 - 2y - 1) = \frac{-\frac{1}{2}x}{3y^2 - 2y - 1}$$

$$\frac{dy}{dx} = \frac{-x}{2(3y^2 - 2y - 1)}$$

PART B

(x, y)

(2, -1)

$$y - y_1 = m(x - x_1)$$

$$y + 1 = m(x - 2)$$

$$y + 1 = 8(x - 2)$$

$$y = 8(x - 2) + 1$$

$$8(1.6 - 2) + 1$$

$$8(-.4) + 1$$

$$-3.2 + 1$$

$$= -2.2$$

~~$$-1(1)^2 - 1(1) + \frac{1}{4}(2)^2$$~~

~~$$-1 - 1 + 1 + \frac{1}{4}(4)$$~~

~~$$-1 + \frac{1}{4}(4)$$~~

~~$$-1 +$$~~

~~$$-2$$~~

~~$$\frac{2(3(1)^2 - 2(1) - 1)}{3 + 2 - 1}$$~~

~~$$3 + 2 - 1$$~~

~~$$2(4) = 8$$~~

Page 14

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

0207358



Q6

NO CALCULATOR ALLOWED

Q6

Answer QUESTION 6 PARTS C and D on this page.

PART C

HOY UUX
 0 2 0
 1 0 1
 1 0 1

$$2(3y^2 - 2y - 1) = 0$$

$$6y^2 - 4y - 2 = 0$$

$$6y^2 - 4y = 2$$

$$\text{points} = \frac{2}{3}$$

$$2y(3y - 2) = 0$$

PART D

$$2y + 2x \frac{dy}{dx} + e^y \frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{2x + e^y}{2x + e^y} = \frac{-2y}{2x + e^y}$$

$$\frac{dy}{dt} \bigg|_{(4,1)} = \frac{-2(1)}{2(4) + e^1} = \frac{-2}{8} = -\frac{1}{4}$$

Question 6

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

NEW for 2025: The question overviews can be found in the *Chief Reader Report on Student Responses* on [AP Central](#).

Sample: 6A

Score: 9 (1-1-1-1-1-1-1-1)

The response earned 9 points: 2 points in part A, 2 points in part B, 2 points in part C, and 3 points in part D.

In part A the response earned **P1** in the third line with a correct implicit differentiation equation. The response earned **P2** with appropriate work in line 4 leading to the correct equation for $\frac{dy}{dx}$ in line 5.

In part B the response earned **P3** with correct simplification of $\frac{-1}{4}$ at the end of line 1. The response would have earned the point in line 1 with $\frac{-2}{2(3(-1)^2 - 2(-1) - 1)}$ without simplification. The response earned **P4** in line 2 on the right with the expression $y \approx \frac{-1}{4}\left(\frac{16}{10}\right) - \frac{1}{2}$. All further simplification presented is correct but unnecessary.

In part C the response earned **P5** with the equation $2(3y^2 - 2y - 1) = 0$ in line 1 on the left. The response earned **P6** with algebra in lines 2 through 7 on the left and the correct value of y in line 1 on the right.

In part D the response earned **P7** in line 1 with the implicit differentiation with respect to t with at most one error. The response earned **P8** in line 1 because the implicit differentiation contains no errors. The response earned **P9** with the correct value of $\frac{-2}{3}$. The response would have earned the point with the unsimplified form $\frac{dy}{dt} = \frac{-6}{9}$.

Sample: 6B

Score: 6 (1-1-1-0-1-1-1-0-0)

The response earned 6 points: 2 points in part A, 1 point in part B, 2 points in part C, and 1 point in part D.

In part A the response earned **P1** with the correct implicit differentiation in line 2. The response earned **P2** with appropriate work in line 3 leading to the correct equation in line 4.

In part B the response earned **P3** in line 1 with $y + 1 = -\frac{1}{4}(x - 2)$, which is a linear approximation with a slope of $-\frac{1}{4}$. The response could also have earned the point with the correct declaration of $\frac{dy}{dx}$ in line 2. The response did not earn **P4** because the correct answer is not given.

In part C the response earned **P5** with the equation $2(3y^2 - 2y - 1) = 0$ in line 1 on the right. The response earned **P6** with subsequent algebra in lines 2 through 6 and with the circled $y = 1$ in line 7 on the right.

Question 6 (continued)

In part D the response earned **P7** with $2x \frac{dy}{dt} \cdot y 2 \frac{dx}{dt} + \frac{1}{y} \frac{dy}{dt} = 0$, which is an attempt at implicit differentiation with respect to t with at most one error. The response did not earn **P8** because the implicit differentiation has one error. The multiplication indicated with the “ $\cdot$ ” should have been addition. The response did not earn **P9** because it did not earn both **P7** and **P8** and is therefore ineligible for **P9**.

Sample: 6C**Score: 3 (1-1-0-0-1-0-0-0-0)**

The response earned 3 points: 2 points in part A, 0 points in part B, 1 point in part C, and 0 points in part D.

In part A the response earned **P1** in the first line with correct implicit differentiation. The response earned **P2** with appropriate work in line 2 leading to the correct equation in line 3.

In part B the response did not earn **P3** because the slope used in the linear approximation is not $-\frac{1}{4}$. The response did not earn **P4** because the linear approximation in line 5 on the left, $8(1.6 - 2) + 1$, is not of the form $-1 + k(-0.4)$ where k is a slope other than $-\frac{1}{4}$ and connected to a declared value of $\frac{dy}{dx}$ at the given point.

In part C the response earned **P5** in the first algebraic line with the equation $2(3y^2 - 2y - 1) = 0$. The response did not earn **P6** because the correct answer is not given.

In part D the response did not earn **P7** because the presented implicit differentiation expression in line 1 has more than one error. The response did not earn **P8** because the presented implicit differentiation expression in line 1 has more than one error. The response did not earn **P9** because it did not earn **P7** and **P8** and is therefore ineligible for **P9**.